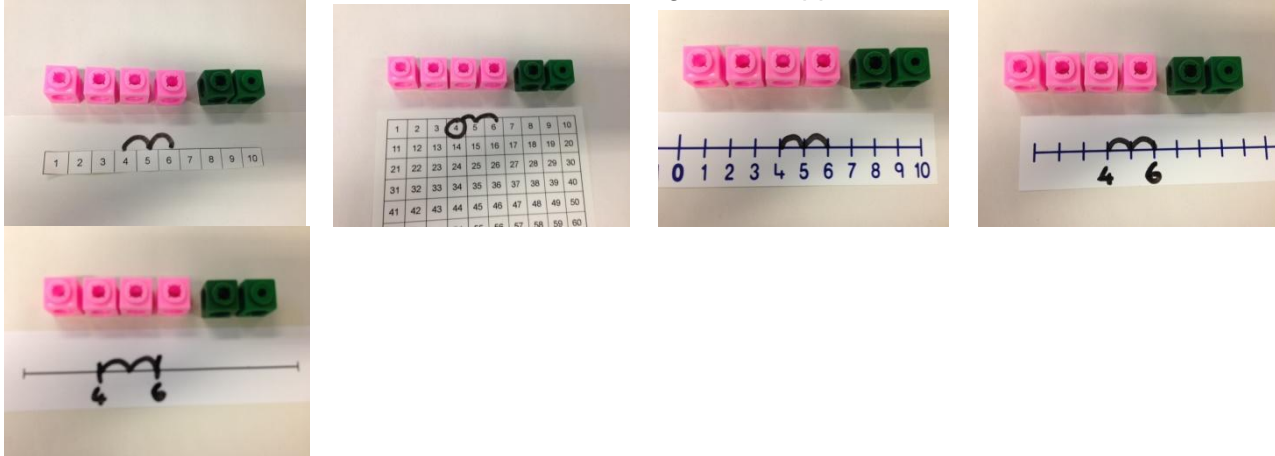


ADDITION CALCULATION GUIDANCE

STAGE 1

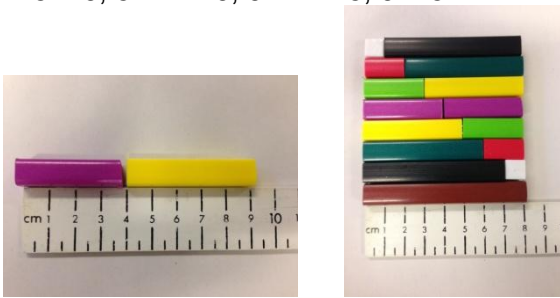
Count on using number tracks / number lines / 100 grids to support.



Develop concept of number bonds, initially to ten and then to 20.

Record related number facts.

e.g. $4 + 5 = 9$, $5 + 4 = 9$, $9 = 4 + 5$, $9 = 5 + 4$



STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as $4 + \square = 9$.

Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $6 + 3 = 9$ (known fact)

$$16 + 3 = 19$$

$$26 + 3 = 29$$

Begin to use understanding of place value and partitioning to carry out addition of one- digit and two-digit numbers

STAGE 3

TU + U

Continue to develop understanding of partitioning and place value and use this to support addition.

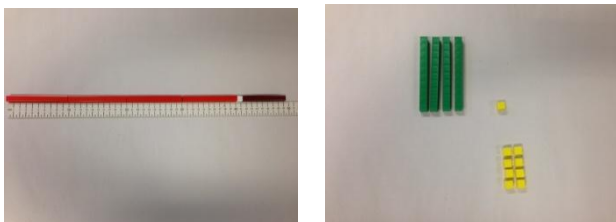
$$41 + 8$$

$$40 + 1 + 8$$

$$40 + 9 = 49$$

Practical apparatus is used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.



When confident with concepts of partitioning and place value, horizontal recording can be replaced with recording in columns with a focus on place value.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 1 \\ + \quad 8 \\ \hline 4 \quad 9 \end{array}$$

STAGE 4

TU + TU

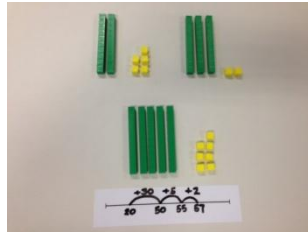
Continue to develop understanding of partitioning and place value and use this to support addition.

$$\begin{array}{l} 25 + 32 \\ 20 + 30 = 50 \\ 5 + 2 = 7 \\ 50 + 7 = 57 \end{array}$$

Practical apparatus is used to support this, as are number tracks /100 squares and number lines. Record the outcomes of calculations in horizontal format.

Where units combine to make totals greater than 10, regroup using partitioning skills

$$\begin{array}{l} 25 + 36 \\ 20 + 30 = 50 \\ 5 + 6 = 11 \\ 50 + 11 = 50 + 10 + 1 = 61 \end{array}$$



Pupils continue to determine when calculations are best carried out using mental strategies.

Horizontal recording can begin to be replaced with recording in columns with a focus on place value. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 2 \quad 5 \\ + 3 \quad 6 \\ \hline 6 \quad 1 \end{array}$$

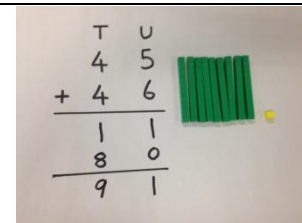
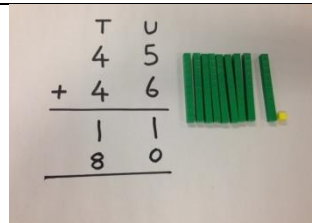
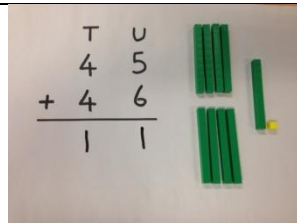
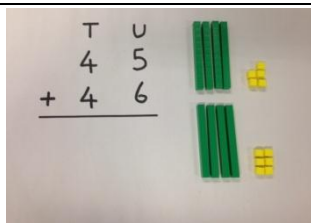
STAGE 5

Continue to determine when calculations are best carried out using mental strategies.

When written methods are more appropriate, continue use of practical apparatus to support, develop an understanding of the formal written method for column addition, initially without and then introducing carrying.

Initially use expanded recording if appropriate to explore concept.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 5 \\ + 4 \quad 6 \\ \hline 1 \quad 1 \\ \hline 8 \quad 0 \\ \hline 9 \quad 1 \end{array}$$



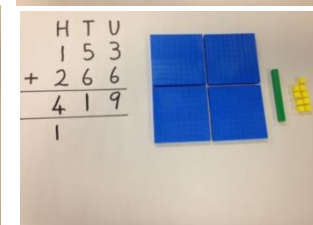
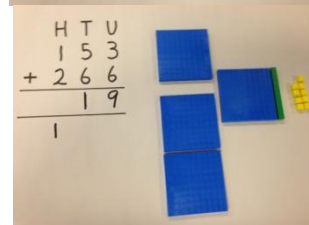
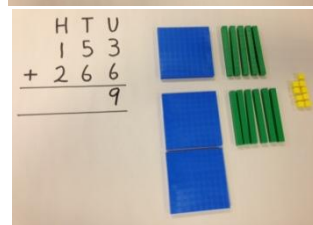
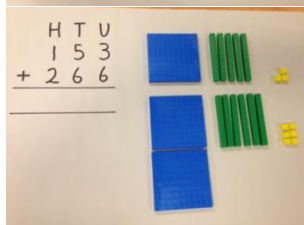
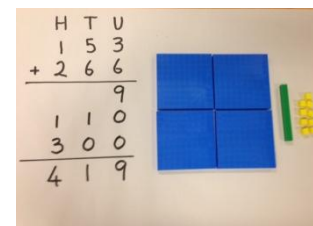
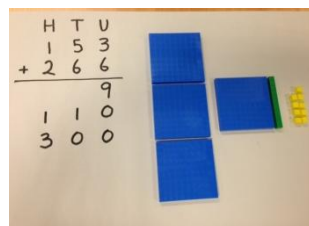
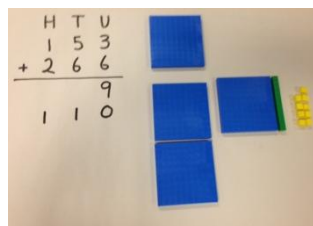
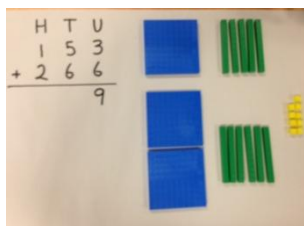
becomes

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 5 \\ + 4 \quad 6 \\ \hline 9 \quad 1 \\ 1 \end{array}$$

STAGE 6

Continue to determine when calculations are best carried out using mental strategies. Extend the standard written method to introduce the hundreds column, initially without and then introducing carrying, initially using expanded recording if appropriate.

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 1 \quad 5 \quad 3 \\ + 2 \quad 6 \quad 6 \\ \hline 9 \\ 1 \quad 1 \quad 0 \\ \hline 3 \quad 0 \quad 0 \\ \hline 4 \quad 1 \quad 9 \end{array}$$



This becomes

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 1 \quad 5 \quad 3 \\ + 2 \quad 6 \quad 6 \\ \hline 4 \quad 1 \quad 9 \\ 1 \end{array}$$

STAGE 7

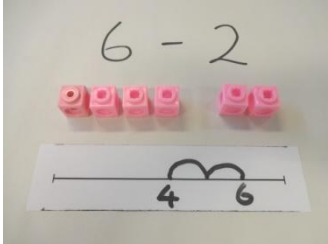
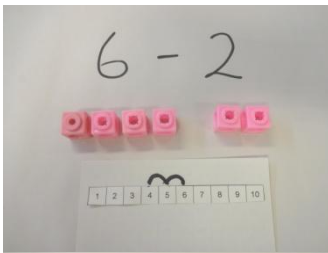
Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to addition of increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

SUBTRACTION CALCULATION GUIDANCE

STAGE 1

Count back using number tracks / number lines / 100 grids to support the development of the concept of subtraction as take away.



Develop subtraction facts initially to ten and then to 20.

Record related number facts (and make links to related addition facts)

e.g. $9 - 4 = 5$, $9 - 5 = 4$

STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as $9 - \square = 5$.

Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $7 - 3 = 4$ (known fact)

$17 - 3 = 14$

$27 - 3 = 24$

Begin to use understanding of place value and partitioning to support subtraction of one-digit and two-digit numbers.

STAGE 3

TU + U

Continue to develop understanding of partitioning and place value and use this to support subtraction.

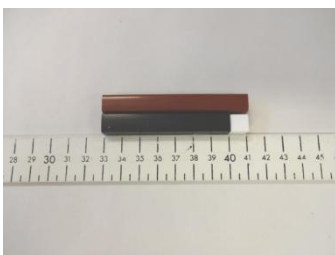
$41 - 8$

$40 - 1 - 7$

$40 - 8 = 33$

Practical apparatus are used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.



$$41 - 8 = 33$$

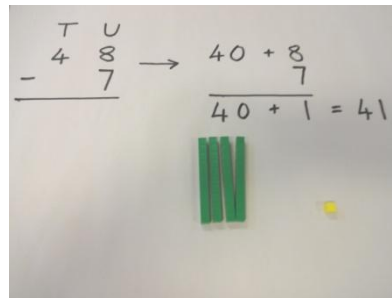
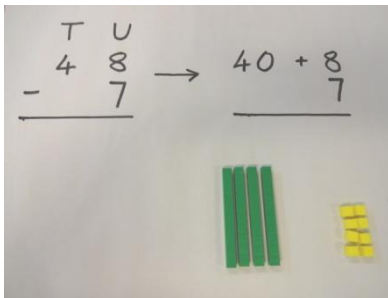
STAGE 4

Pupils continue to determine when calculations are best carried out using mental strategies.

Horizontal recording can begin to be replaced with recording in columns with a focus on place value. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

No exchange

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 8 \\ - \quad 7 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 8 \\ - \quad 7 \\ \hline 40 + 1 = 41 \end{array}$$

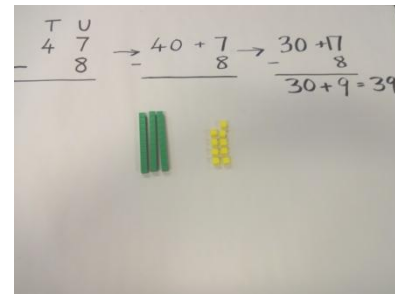
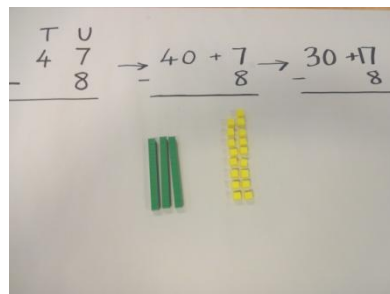
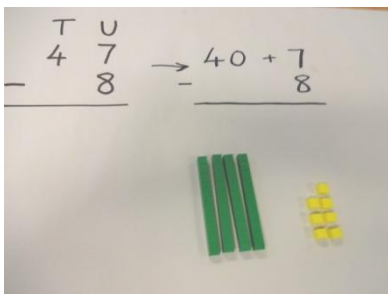


becomes

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 8 \\ - \quad 7 \\ \hline 4 \quad 1 \end{array}$$

Exchange

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 7 \\ - \quad 8 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 7 \\ - \quad 8 \\ \hline \end{array} \longrightarrow \begin{array}{r} 30 + 17 \\ - \quad 8 \\ \hline 30 + 9 = 39 \end{array}$$



becomes

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 7 \\ - \quad 8 \\ \hline 4 \quad 1 \end{array} \longrightarrow \begin{array}{r} \text{T} \quad \text{U} \\ 3 \quad 17 \\ - \quad 8 \\ \hline 4 \quad 1 \end{array}$$

STAGE 5

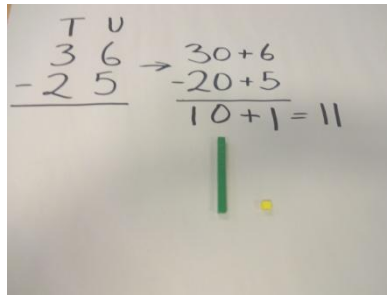
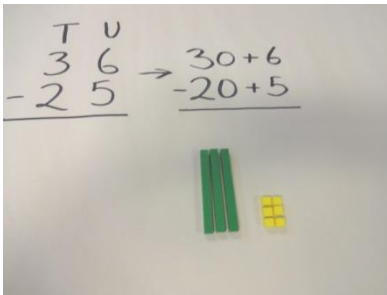
TU + TU

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

No exchange

$$\begin{array}{r} \text{T} \quad \text{U} \\ 3 \quad 6 \\ - \quad 2 \quad 5 \\ \hline \end{array} \longrightarrow \begin{array}{r} 30 + 6 \\ - 20 + 5 \\ \hline 10 + 1 = 11 \end{array}$$

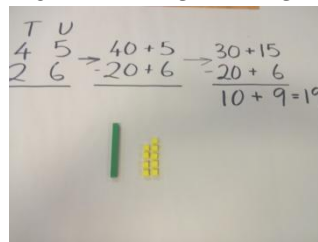
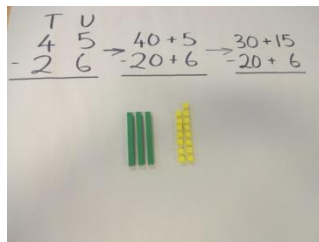
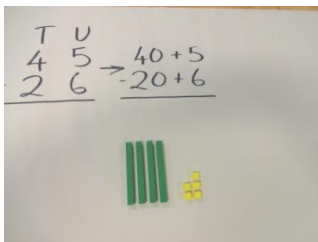


becomes

$$\begin{array}{r} \text{T} \quad \text{U} \\ 3 \quad 6 \\ - \quad 2 \quad 5 \\ \hline 1 \quad 1 \end{array}$$

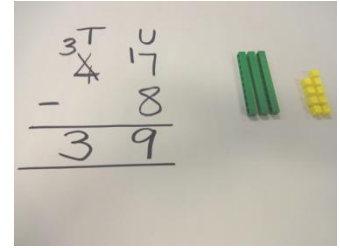
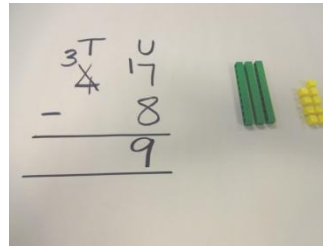
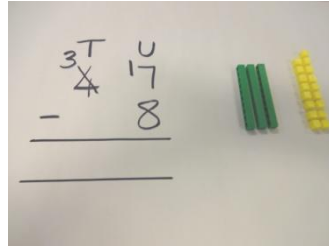
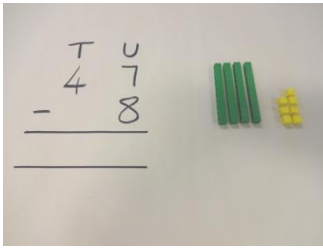
Exchange

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 5 \\ - \quad 2 \quad 6 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 5 \\ - 20 + 6 \\ \hline \end{array} \quad \begin{array}{r} 30 + 15 \\ - 20 + 6 \\ \hline 10 + 9 = 19 \end{array}$$



becomes

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 5 \\ - 2 \quad 6 \\ \hline \end{array} \longrightarrow \begin{array}{r} \text{T} \quad \text{U} \\ 3 \quad 15 \\ - 2 \quad 6 \\ \hline 3 \quad 9 \end{array}$$



STAGE 6

HTU - HTU

Continue to determine when calculations are best carried out using mental strategies.

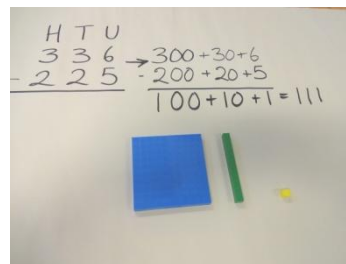
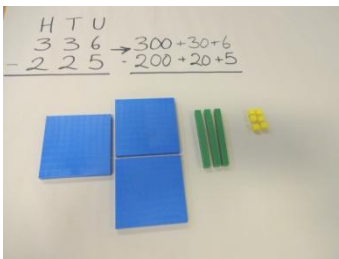
Develop use of the formal written method. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

Explore how the process relates to numbers with zeros as place holders.

No exchange

Using an expanded method of recording if appropriate before moving to formal method

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 3 \quad 3 \quad 6 \\ - 2 \quad 2 \quad 5 \\ \hline \end{array} \longrightarrow \begin{array}{r} 300 + 30 + 6 \\ - 200 + 20 + 5 \\ \hline 100 + 10 + 1 = 111 \end{array}$$



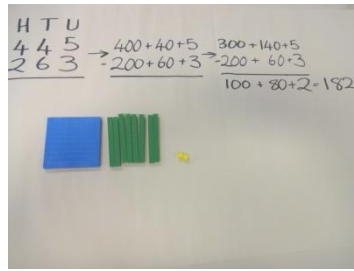
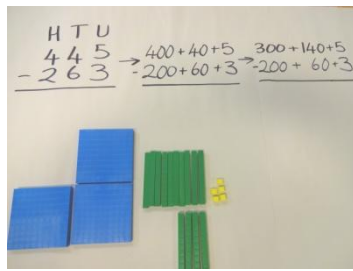
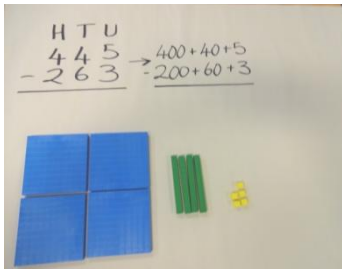
becomes

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 3 \quad 3 \quad 6 \\ - 2 \quad 2 \quad 5 \\ \hline 1 \quad 1 \quad 1 \end{array}$$

Exchange

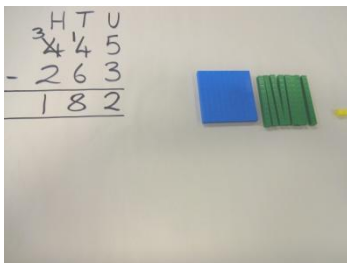
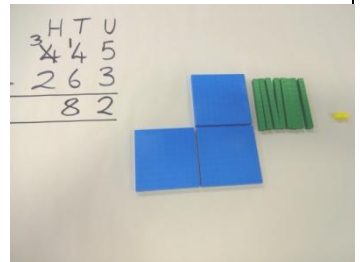
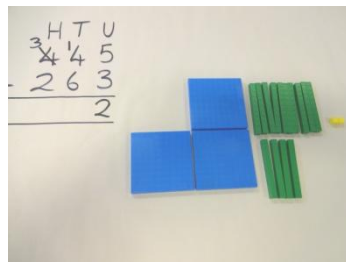
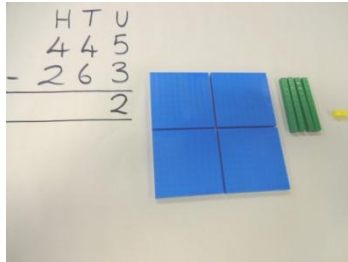
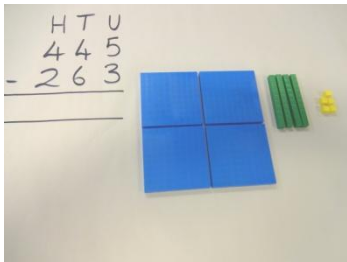
$$\begin{array}{r} \text{H T U} \\ 445 \\ - 263 \\ \hline \end{array} \longrightarrow \begin{array}{r} 400 + 40 + 5 \\ - 200 + 60 + 3 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 300 + 140 + 5 \\ - 200 + 60 + 3 \\ \hline 80 + 2 \end{array} \longrightarrow \begin{array}{r} 300 + 140 + 5 \\ - 200 + 60 + 3 \\ \hline 100 + 80 + 2 = 182 \end{array}$$



becomes

$$\begin{array}{r} \text{H T U} \\ \overset{3}{\cancel{4}} 4 5 \\ - 2 6 3 \\ \hline 1 8 2 \end{array}$$



Exchange with place holders

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 4 \quad 0 \quad 5 \\ - 2 \quad 6 \quad 9 \\ \hline \end{array} \quad \begin{array}{l} \longrightarrow \\ \longleftarrow \end{array} \quad \begin{array}{r} 400 + 0 + 5 \\ - 200 + 60 + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 300 + 100 + 5 \\ - 200 + 60 + 9 \\ \hline \end{array} \quad \begin{array}{r} 300 + 90 + 15 \\ - 200 + 60 + 9 \\ \hline 100 + 30 + 3 = 133 \end{array}$$

becomes

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ \overset{3}{4} \quad \overset{1}{0} \quad \overset{1}{5} \\ - 2 \quad 6 \quad 9 \\ \hline 1 \quad 3 \quad 3 \end{array}$$

STAGE 7

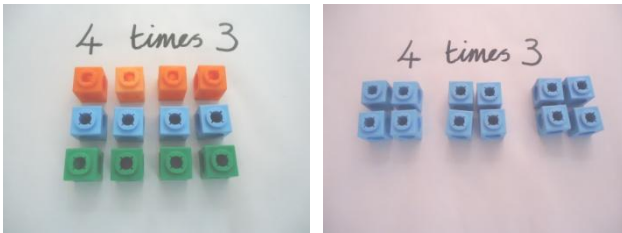
Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to subtraction of increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

MULTIPLICATION CALCULATION GUIDANCE

STAGE 1

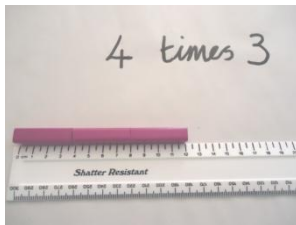
Develop multiplication as repeated grouping (repeated addition of sets of the same size) using practical apparatus and diagrams.



STAGE 2

Develop an understanding of multiplication using arrays and number lines showing repeated groups.

Use number lines to show repeated grouping (repeated addition of sets of the same size).



STAGE 3

Develop the use of x and = symbols to record calculations horizontally.

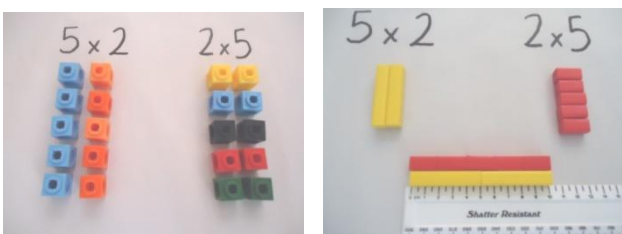
Use arrays and other practical apparatus to illustrate commutativity (that multiplication calculations can be carried out in any order) e.g. 2×5 arrives at the same product as 5×2 .

Begin to derive new facts from known facts

e.g. $3 \times 2 = 6$ (known fact)

$30 \times 2 = 60$

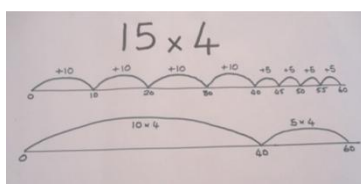
$300 \times 2 = 600$ etc.



STAGE 4

Begin to use understanding of place value and partitioning to carry out multiplication of two-digit by one-digit numbers

$$\begin{array}{r} 15 \times 4 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$



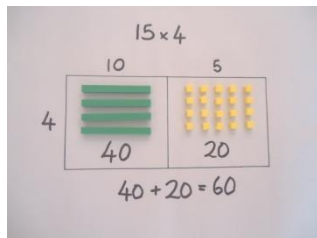
$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

$$40 + 20 = 60$$

Use grid approaches to illustrate as appropriate using practical apparatus to support.

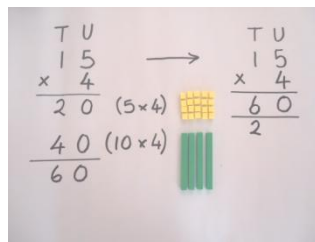
$$4 \begin{array}{|c|c|} \hline 10 & 5 \\ \hline 40 & 20 \\ \hline \end{array}$$



Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 1 \quad 5 \\ \times \quad 4 \\ \hline 2 \quad 0 \quad (5 \times 4) \\ 4 \quad 0 \quad (10 \times 4) \\ \hline 6 \quad 0 \end{array} \longrightarrow$$

$$\begin{array}{r} \text{T} \quad \text{U} \\ 1 \quad 5 \\ \times \quad 4 \\ \hline 6 \quad 0 \\ 2 \end{array}$$

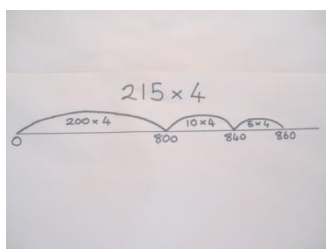


STAGE 5

Extend written approaches to HTU x U, then to ThHTU x U

Illustrate using partitioning approaches as required

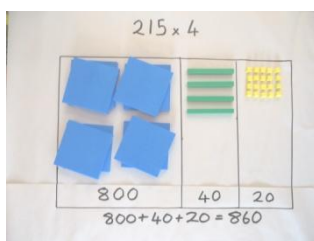
$$\begin{array}{c} 215 \times 4 \\ \swarrow \quad \downarrow \quad \searrow \\ 200 \quad 10 \quad 5 \end{array}$$



$$\begin{aligned} 200 \times 4 &= 800 \\ 10 \times 4 &= 40 \\ 5 \times 4 &= 20 \\ 800 + 40 + 20 &= 860 \end{aligned}$$

Illustrate using grid approaches as required

$$4 \begin{array}{|c|c|c|} \hline 200 & 10 & 5 \\ \hline 800 & 40 & 20 \\ \hline \end{array}$$



$$800 + 40 + 20 = 860$$

Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 1 \quad 1 \quad 5 \\ \times \quad 4 \\ \hline 8 \quad 0 \quad 0 \quad (5 \times 4) \\ 4 \quad 0 \quad 0 \quad (10 \times 4) \\ 8 \quad 0 \quad 0 \quad (200 \times 4) \\ \hline 8 \quad 6 \end{array} \longrightarrow$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 2 \quad 1 \quad 5 \\ \times \quad 4 \\ \hline 8 \quad 6 \quad 0 \\ 2 \end{array}$$

STAGE 6

Extend written approaches to HTU x TU and ThHTU x TU

Illustrate using grid approaches as required

$$26 \times 13$$

	20	6	
10	200	60	
3	60	18	

$$200 + 60 + 60 + 18 = 338$$

Develop expanded recording in columns and then move to formal written method of long multiplication, using practical apparatus to support as required.

	H	T	U		H	T	U
		2	6			2	6
x		1	3	→	x	1	3
		1	8			7	8
			(6 x 3)			1	
		6	0			2	6
			(20 x 3)			3	8
		6	0			1	
			(6 x 10)			2	6
	2	0	0			3	8
			(10 x 20)			1	
	3	3	8				
			1				

$$226 \times 13$$

	200	20	6	
10	2000	200	60	
3	600	60	18	

$$2000 + 600 + 200 + 60 + 60 + 18 = 2938$$

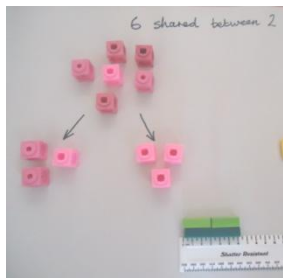
Develop expanded recording in columns and then move to formal method of long multiplication, using practical apparatus to support as required.

	H	T	U		H	T	U
	2	2	6			2	6
x		1	3	→	x	1	3
		1	8			6	7
			(6 x 3)			1	
		6	0			2	6
			(20 x 3)			2	6
	6	0	0			2	9
			(200 x 3)			3	8
		6	0			1	
			(6 x 10)			2	9
	2	0	0			3	8
			(20 x 10)			1	
	2	0	0				
			(200 x 10)				
	2	9	3				
			8				
			1				

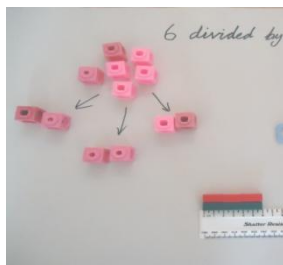
DIVISION CALCULATION GUIDANCE

STAGE 1

Develop division as sharing



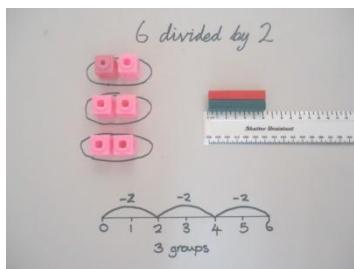
Develop division as repeated grouping (repeated subtraction of sets of the same size) using practical apparatus and diagrams.



STAGE 2

Develop an understanding of division using arrays and number lines showing repeated groups

Use number lines to show repeated grouping (repeated subtraction of sets of the same size)



STAGE 3

Develop the use of \div and $=$ symbols to record calculations horizontally

Use arrays and other practical apparatus to illustrate making of repeated groups

Begin to derive new facts from known facts

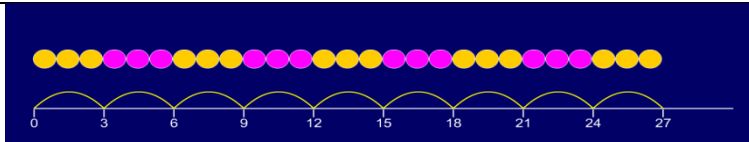
e.g. $6 \div 2 = 3$ (known fact)

$60 \div 2 = 30$

$600 \div 2 = 300$

Begin to carry out division of two-digit by one-digit numbers, first without remainders, then introducing remainders, illustrating this using informal methods first if required.

$27 \div 3$

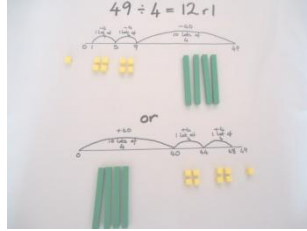
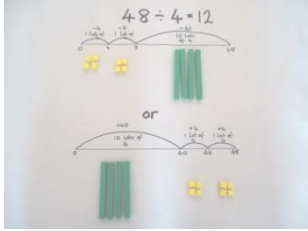


STAGE 4

Division using larger multiples of the divisor, first with no remainders, then with remainders

$$48 \div 4$$

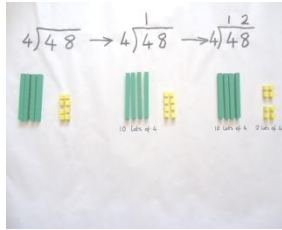
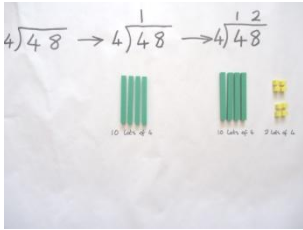
$$49 \div 4$$



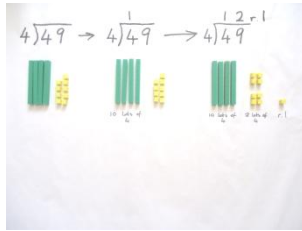
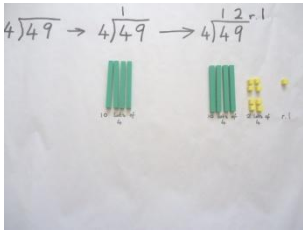
STAGE 5

Move to develop the standard method for short division, first with no remainders, then with remainders

$$48 \div 4$$



$$49 \div 4$$



STAGE 6

Extend written calculation methods to HTU \div U, then to ThHTU \div U, first with no remainders, then with remainders, illustrating this using informal methods first if required.

No carrying forward required

$$448 \div 4$$

(as above, but with additional hundreds column)

No carrying forward required, but with remainders

$$449 \div 4$$

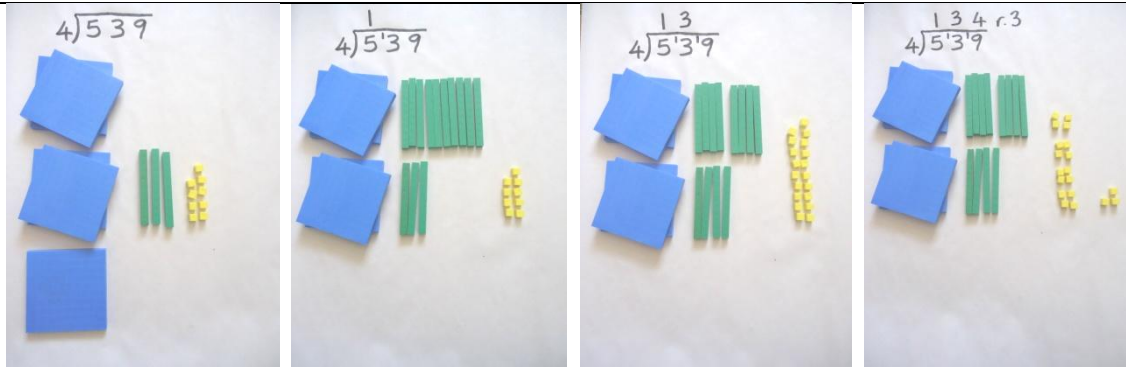
(as above, but with additional hundreds column)

Carrying forward required

$$536 \div 4$$

Carrying forward required, but with remainders

$$539 \div 4$$



STAGE 7

Extend written approaches to the formal method of long division when dividing by two-digit numbers, illustrating this using informal methods first if required.

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{300} \quad (15 \times 20) \\
 132 \\
 \underline{120} \quad (15 \times 8) \\
 12
 \end{array}$$

becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{30} \quad \downarrow \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$